Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

MATHEMATICS: SPECIALIST

UNITS 3A AND 3B

FORMULA SHEET
2013

This document is valid for teaching and examining until 31 December 2013.

Vectors

Magnitude: $|(a_1, a_2)| = \sqrt{a_1^{\,2} + a_2^{\,2}}$

Dot product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta = a_1 b_1 + a_2 b_2$

Triangle inequality: $|\mathbf{a} + \mathbf{b}| \le |\mathbf{a}| + |\mathbf{b}|$

Vector equation of a line in the plane:

- one point and the slope: $\mathbf{r} = \mathbf{r}_1 + \lambda \mathbf{l}$
- two points: $\mathbf{r} = \mathbf{r}_1 + \lambda(\mathbf{r}_2 - \mathbf{r}_1)$
- normal: $\mathbf{r} \bullet \mathbf{n} = c$

Vector equation of a circle in the plane: $|\mathbf{r} - \mathbf{d}| = \rho$

Trigonometry

In any triangle ABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\text{Area} = \frac{1}{2} ab \sin C$$

In a circle of radius r, for an arc subtending angle θ (radians) at the centre:

Length of arc $= r\theta$

Area of segment $= \frac{1}{2} r^2 (\theta - \sin\theta)$

Area of sector $= \frac{1}{2} r^2 \theta$

Identities:

$$\cos^2\theta + \sin^2\theta = 1$$

$$\cos(\theta \pm \varphi) = \cos\theta \cos\varphi \mp \sin\theta \sin\varphi$$

$$\sin(\theta \pm \varphi) = \sin\theta \cos\varphi \pm \cos\theta \sin\varphi$$

$$\tan(\theta \pm \varphi) = \frac{\tan\theta \pm \tan\varphi}{1 \mp \tan\theta \tan\varphi}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

See next page

Exponentials and logarithms

For a, $b > 0$ and m, n real:

$$a^m a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n}$$

$$a^0 = 1 \qquad a^{-n} = \frac{1}{a^n}$$

$$(a^m)^n = a^{mn} \qquad (ab)^m = a^m b^m$$

For $a > 0$ and m an integer and n a positive integer:

$$a^{\frac{1}{n}} = \sqrt[n]{a} \qquad a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

For a, y, m, n positive and real and k real:

$$1 = a^0 \Leftrightarrow \log_a 1 = 0 \qquad y = a^x \Leftrightarrow \log_a y = x$$

$$\log_a(mn) = \log_a m + \log_a n \qquad a = a^1 \Leftrightarrow \log_a a = 1$$

$$\log_a(m^k) = k \log_a m$$

Measurement

Circle: $C = 2\pi r = \pi D$, where C is the circumference, r is the radius and D is the diameter
$A = \pi r^2$, where A is the area

Triangle: $A = \frac{1}{2}bh$, where b is the base and h is the perpendicular height

Parallelogram: $A = bh$

Trapezium: $A = \frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides

Prism: $V = Ah$, where V is the volume, A is the area of the base

Pyramid: $V = \frac{1}{3}Ah$

Cylinder: $S = 2\pi rh + 2\pi r^2$, where S is the total surface area
$V = \pi r^2 h$

Cone: $S = \pi rs + \pi r^2$, where s is the slant height
$V = \frac{1}{3}\pi r^2 h$

Sphere: $S = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$

Functions

Differentiation: If $f(x) = y$ then $f'(x) = \frac{dy}{dx}$

If $f(x) = x^n$ then $f'(x) = nx^{n-1}$

If $f(x) = e^x$ then $f'(x) = e^x$

If $f(x) = \ln x$ then $f'(x) = \frac{1}{x}$

Product rule: If $y = f(x)\,g(x)$ then $y' = f'(x)\,g(x) + f(x)\,g'(x)$

or

If $y = uv$ then $\frac{dy}{dx} = \frac{du}{dx}v + u\frac{dv}{dx}$

Quotient rule: If $y = \frac{f(x)}{g(x)}$ then $y' = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$

or

If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$

Chain rule: If $y = f(g(x))$ then $y' = f'(g(x))\,g'(x)$

or

If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Quadratic function: If $y = ax^2 + bx + c$ and $y = 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for $x \in \mathrm{C}$

Piecewise-defined functions:

Absolute value function: $|x| = \begin{cases} x, & \text{for } x \geq 0 \\ -x, & \text{for } x < 0 \end{cases}$

Sign function: $\operatorname{sgn}(x) = \begin{cases} 1, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -1, & \text{for } x < 0 \end{cases}$

Greatest integer function: $\operatorname{int}(x)$ = greatest integer $\leq x$ for all x

Note: Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.